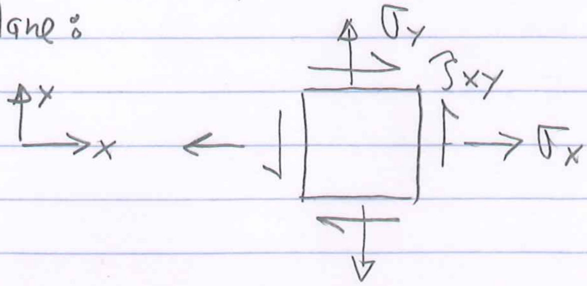
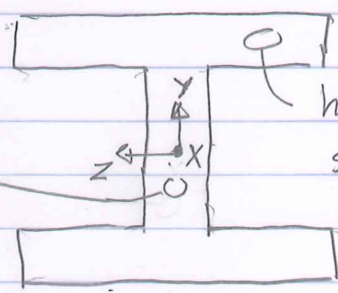
25. Stress transformation: plane stress

Recall that plane stress is a 2-d state of stress. For the x-y plane:



$\sigma_z = 0$
$\tau_{yz} = 0$
$\tau_{xz} = 0$

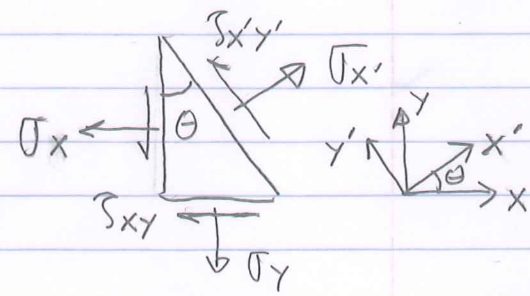
Plane stress occurs quite often. For an I-beam under bending and shear:



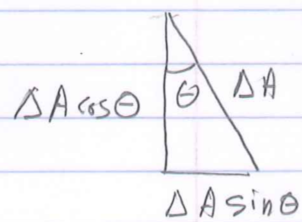
have σ_x, τ_{xy} (plane stress in the x-y plane)

have σ_x, τ_{xz} (plane stress in the x-z plane)

Stress transformation means that we want to express $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$ in terms of σ_x, σ_y, τ_{xy} and θ, where θ orients the x'-y' coordinates with respect to x-y.



The differential element shown has face areas



Write $\Sigma F_{x'} = 0$ and $\Sigma F_{y'} = 0$ for the differential element.

$$\sigma_{x'}\Delta A - \sigma_x \cos\theta \cdot \Delta A \cos\theta - \tau_{xy}\sin\theta \cdot \Delta A \cos\theta$$
$$- \sigma_y \sin\theta \cdot \Delta A \sin\theta - \tau_{xy}\cos\theta \cdot \Delta A \sin\theta = 0$$

$$\tau_{x'y'}\,\Delta A + \sigma_x \sin\theta \cdot \Delta A \cos\theta - \tau_{xy} \cos\theta \cdot \Delta A \cos\theta$$
$$- \sigma_y \cos\theta \cdot \Delta A \sin\theta + \tau_{xy} \sin\theta \cdot \Delta A \sin\theta = 0$$

After some algebra, we get

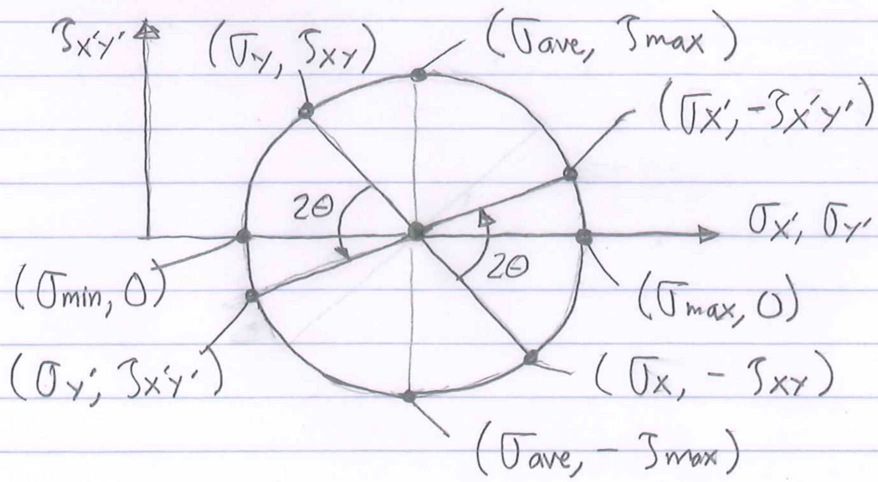
$$\sigma_{x'} = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\tfrac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Substitution of $\theta + 90°$ into the $\sigma_{x'}$ expression yields $\sigma_{y'}$ as

$$\sigma_{y'} = \tfrac{1}{2}(\sigma_x + \sigma_y) - \tfrac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta - \tau_{xy}\sin 2\theta$$

These expressions can be conveniently plotted as Mohr's circle for plane stress.



where $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$. The center of the circle is at $(\sigma_{ave}, 0)$ and the radius of the circle is

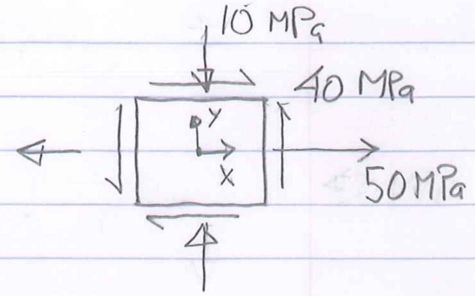
$$R = \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}}$$

σ_{max} and σ_{min} are the principal stresses. They occur on perpendicular planes, and are given by

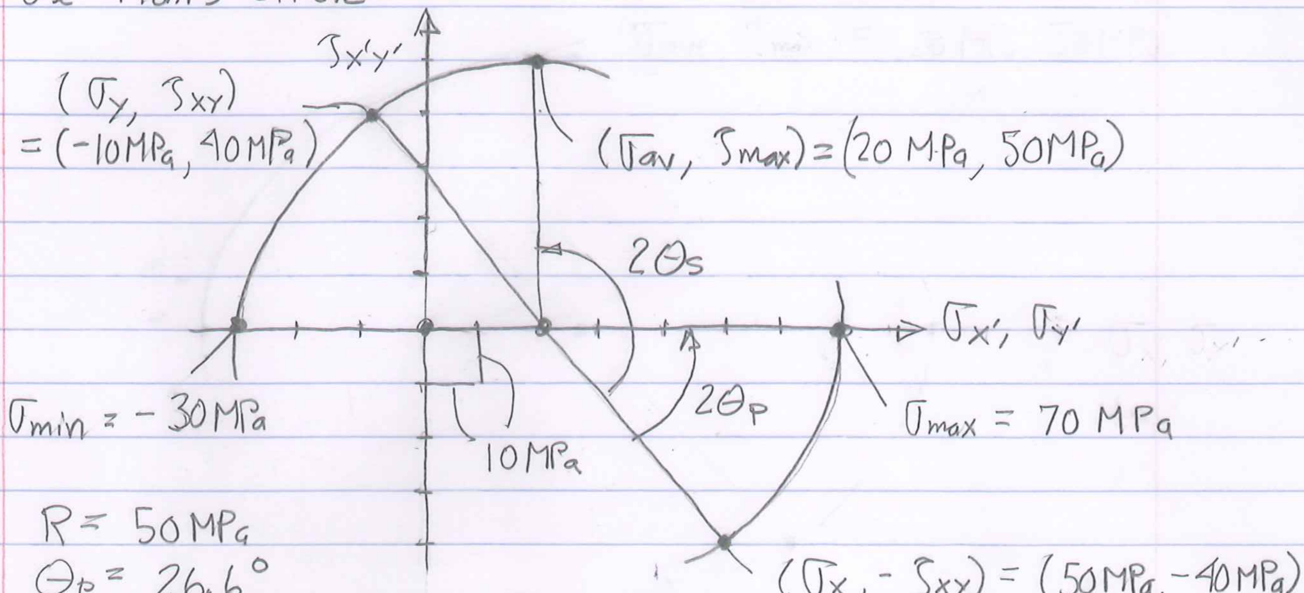
$$\sigma_{max}, \sigma_{min} = \sigma_{ave} \pm R$$

Also, the maximum shear stress is $\tau_{max} = R$.

Example: Find τ_{max}, σ_{max}, σ_{min}
for $\sigma_x = 50$ MPa
$\sigma_y = -10$ MPa
$\tau_{xy} = 40$ MPa

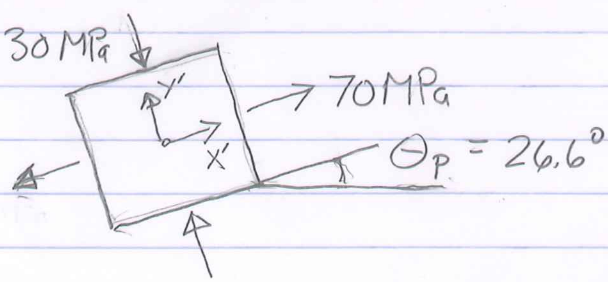


Use Mohr's circle



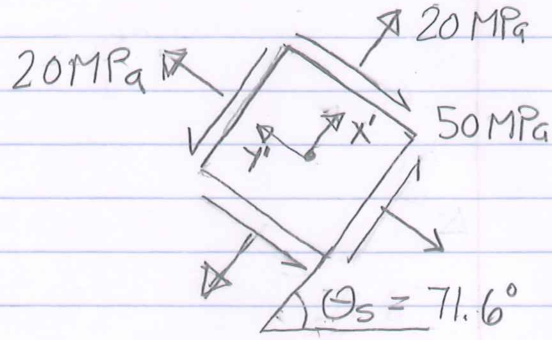
$R = 50$ MPa
$\theta_p = 26.6^\circ$
$\theta_s = 71.6^\circ$

principal stresses



$\sigma_{x'} = 70$ MPa, $\sigma_{y'} = -30$ MPa
$\tau_{x'y'} = 0$

maximum shear stress



$\sigma_{x'} = \sigma_{y'} = 20$ MPa
$\tau_{x'y'} = -50$ MPa

The extreme stress values τ_{max}, σ_{min} and σ_{max} may not be the true extreme values of stress for any arbitrarily oriented plane. This is discussed in the next section.